Heat transfer in temperature-dependent non-Newtonian flow

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Abstract
Flow and thermal convection in heat exchangers can be affected by variations in viscosity. In circular pipes the significance of this effect was expressed by Sieder and Tate’s empirical formula. Similar correction can be derived for other heat transfer problems. Exact solutions for heat transfer during the non-isoviscous laminar flow of Newtonian and non-Newtonian liquids in circular and flat ducts under conditions of constant wall temperature or constant wall heat flux are analyzed. Simple generalized procedures for heat transfer coefficient and friction estimate are recommended.

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1. Introduction
Thermal convection under non-isoviscous conditions is retarded if the viscosity near the wall increases and is intensified if the viscosity decreases. The first systematic investigation of the problem was conducted by Sieder and Tate (ST) [1] who studied heat transfer in a tubular heat exchanger at a constant wall temperature. They recommended expressing the effect of viscosity variation by a simple correction:

$\frac{Nu}{Nu_{M}} = \left( \frac{\bar{\mu}_B}{\bar{\mu}_W} \right)^{0.14}.$

(1)

The power 0.14 was found by regression of the experimental data. $Nu_M$ and $Nu_{M,\star}$ are the actual Nusselt number and $Nu_{M,\star}$, is the value corresponding to the case of temperature-independent (isoviscous) flow. The viscosities $\bar{\mu}_B$, $\bar{\mu}_W$ were related to the average temperatures of the bulk and wall, respectively, calculated simply from the values at the entrance and exit of the heat transfer section,

$\bar{T}_B(x) = \frac{T_B(0) + T_B(x)}{2},$

(2)

$\bar{T}_W(x) = \frac{T_W(0) + T_W(x)}{2}.$

(3)

The formula (1) was obtained from a limited number of heat transfer experiments.

$\frac{Nu}{Nu_{M}} = Nu_{M,\star} \left( \frac{\bar{\mu}_B}{\bar{\mu}_W} \right)^{0.14}$

(4)

may also depend on the heat exchanger length, on the viscosity ratio, and generally on the boundary conditions describing the shape of the heat exchanging equipment.

Liquid viscosity usually increases exponentially with rising temperature. One suitable model is represented by the formula

$\mu(T) = \mu_0 \exp \left( -\frac{T - T_0}{T_V} \right),$

(5)

which is somewhat related to the Arrhenius-type semitheoretical equation and fits well with the data for most fluids. It contains two constants with a simple physical meaning: $\mu_0$ is the viscosity at the temperature $T_0$, $T_V$ is a material property and it is equal to the temperature difference corresponding to the viscosity ratio 1/exp. The effect of variable viscosity in systems with characteristic temperature difference $T_W - T_0$ can be expressed by the dimensionless number

$\psi = -\ln \left( \frac{\mu_W}{\mu_0} \right) \frac{T_W - T_0}{T_V},$

(6)

which is sometimes called the Pearson number [2].
with symmetric heat transfer on both walls, where $B$ is equal to the channel width $B = W$ and the flat channel with symmetric heat transfer on both walls, where $B = W/2$.

In other applications we can also find some modified definitions of ST correlation, e.g.

1. Numerical solution

We have developed an efficient numerical procedure, which enables us to solve with great accuracy the heat transfer problem within a large range of variables. Moreover, we have investigated analytical solutions of some asymptotic problems of non-isoviscous flow, which is useful for checking the accuracy of numerical results. We have found that the very common neglecting of the radial convection term leads to serious errors in some non-isoviscous flow solutions.

The temperature field, shown schematically in Fig. 2, can be subdivided into three regions. In the thermal entry region, temperature changes occur in a narrow thermal boundary layer, $A$, close to the wall, while in region $B$ beyond this layer the wall effect can be neglected. Our numerical approach uses a non-equidistant finite difference scheme in...
region A. For any value of the axial coordinate, x, we have a definite number of equidistant intervals Δy, covering just the layer A and the intervals Δy follows the condition of numerical stability Δx ∼ Δy. The temperature field in region B is trivial and the related velocity field can be solved analytically. For a fully developed temperature profile, C, we employ the standard equidistant grid.

Problems solved:
1. Geometry configuration and the boundary conditions:
   - Circular tube (R);
   - Flat duct, symmetric heat transfer on both walls (F2);
   - Flat duct with asymmetric thermal conditions:
     - Thermal exchange at one wall, the other being insulated (F1).
2. Hydrodynamically developed laminar flow:
   - Newtonian liquids (N);
   - Power law fluids (NN).
3. Boundary conditions:
   - Constant wall temperature (CWT);
   - Constant wall heat flux (CHF).

Full scale numerical solutions were obtained for (R) and (F2) cases. The power law indexes were assumed to have 1/n integers. This makes some computations easier, however it is not mandatory. The (F1) problem was solved for (N) liquids.

Adequacy of the numerical results has been checked by comparison with the results or independent asymptotic solution of the problem: For the thermal entrance region, z → 0, the asymptotic boundary layer problem is expressed by ordinary differential equations. The solution for CWT was obtained in the form of simple algebraic functions by the solution of the thermal boundary layer problem with a linear temperature profile. This method was presented for z → 0 [12,13] and we solved the problem also for finite z at ϑ → 0 [14].

Full numerical solution with 200 MHz PC using 20 equidistant intervals Δy, covering layers A and C, yield results for one temperature field with an accuracy 1% in 30 s, for 60 intervals (as used in most of our computations) it took 20 min on average and error greater than 0.01% in any parameter was not found.

3. Results and discussion
3.1. Isoviscous heat transfer

The Nusselt number for circular tubes uses a tube diameter as a characteristic length and the value 2B is also introduced to the generalized definition:

\[ Nu_i = \frac{2 Bh_i}{\lambda} \quad (i = M, X) \]  

For heat transfer in ducts, two significant asymptotes can be found. For z → 0, the Pigford [15] type asymptote gives

\[ \lim_{z \to 0} Nu^*_M = 1.0767 z^{-1/3} \left( \frac{\gamma B}{U} \right)^{1/3} \]  
\[ \lim_{z \to 0} Nu^*_X = 2 \lim_{z \to 0} Nu^*_M \]  

and

For CWT:

\[ \lim_{z \to 0} Nu^*_M = 1.3020 z^{-1/3} \left( \frac{\gamma B}{U} \right)^{1/3} \]  
\[ \lim_{z \to 0} Nu^*_X = 2 \lim_{z \to 0} Nu^*_M \]  

Dimensionless wall shear rate in Newtonian and power law fluids is

For R:

\[ \frac{\gamma B}{U} = \frac{3n + 1}{n} \]  

For F2:

\[ \frac{\gamma B}{U} = \frac{2(2n + 1)}{n} \]  

For F1:

\[ \frac{\gamma B}{U} = \frac{2(2n + 1)}{n} \]  

Ratio of actual values of \( Nu^*_M \) for finite z to the extrapolated ones for z → 0, according to (16) and (18), is interpreted in Fig. 3 by the coefficient A, which is

For CWT:

\[ A_X(z) = Nu^*_X z^{1/3} \left( \frac{\gamma B/U}{0.0767} \right)^{1/3} \]  

and

For CHF:

\[ A_X(z) = Nu^*_X z^{1/3} \left( \frac{\gamma B/U}{1.3020} \right)^{1/3} \]
Fig. 3. Ratio (23), (24) of the local Nusselt number to the value predicted by Lévêque at $\Psi \to 0$ and $n = 1$.

Fig. 4. Local Nusselt numbers at $\Psi = 0$ and $n = 1$ for nearly developed heat transfer.

For $z < 0.1$ where $A_X \approx 1$, the Pigford [15] asymptote (16)–(19) is a suitable approximation.

For higher $z$, the local Nusselt numbers approaches asymptotic limiting values for fully developed heat transfer, as shown in Fig. 4. Related asymptotic values are plotted in Table 1.

3.2 Effect of temperature changes of viscosity to the heat transfer

Different approaches to the heat transfer corrections are presented for $n = 1$, and $Y \to 0$ in Fig. 5.

Original ST correction gives different values for CWT and CHF problems, while the $\alpha_X$ and $\alpha_M$ are quite insensitive

Table 1

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<th>$\alpha$</th>
<th>R</th>
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<td>CHF</td>
<td>CWT</td>
<td>CHF</td>
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</table>

Fig. 5. Different approaches $\alpha_X$, $\alpha_M$, $\alpha_{ST}$ to the heat transfer corrections for $n = 1$ and $Y \to 0$. 
to the boundary conditions. For the thermal entrance region, $z \to 0$ and generally for CHF $\frac{\Nu_X}{\Nu_M}$ and $\alpha_X = \alpha_M$. However, the value $\frac{\Nu_X}{\Nu_M}$ gives better information than $\Nu_M$ for the developed heat transfer regime at $z \to \infty$ with CWT and this explains why all following results will be demonstrated by using $\alpha_X$.

Effect of non-isoviscousness is shown in the plot of $\alpha_X(\Psi)$ for Newtonian liquids at different $z$, and different boundary conditions in Fig. 6.

This function can be independently obtained by solving ordinary differential equations available for CWT at $z \to 0$ [11], and for CHF at $z \to \infty$. It is apparent that $\alpha_X(\Psi)$ is close to a constant, and therefore a single value $\alpha_X$ could be accepted in the most of practical calculations. Henceforth, the effect of $z$, boundary conditions and flow index $n$, will be studied for $\Psi \to 0$ only.

The effect of $z$ at various boundary conditions is demonstrated for Newtonian liquids in Fig. 6. For $z \to 0$ it is close to the approximate analytic solution [12,13] giving $\lim_{z \to 0} \alpha_X = 1/4$. For the range $0.005 < z < 0.1$, which is the most common for heat exchangers $\alpha_X$ is close to the experimental value 0.14 by Sieder and Tate [1].
Non-Newtonian liquid behavior is presented as a function $a_X(n)$ in Fig. 7, where the viscosity ratio is based on the values at a constant shear stress, Eq. (15). In such case we have a common asymptote at $z \rightarrow 0$: Metzner et al.’s [6] original idea that the viscosities at the constant shear rate should be taken into account, predicts $a(n) = n\alpha(1)$. In all cases this results in the underestimation of the value $a_X(n)$, though the error for higher values $z$ is not serious.

3.3. Pressure drop in non-isoviscous flow

Average local viscosity $\mu_P$ (or $\eta_P$) is defined as the value of viscosity of isothermal flow of a given fluid which for a given $x$ predicts the identical pair of values, $d\rho/dx$ and $U$ as the actual non-isoviscous flow. For a given liquid this viscosity $\mu_P$ (or $\eta_P$) corresponds to the pressure-averaged temperature $T_P$ introduced by Wichterle [10,11]. The value $T_P$ is between $T_W$ and $T_M$ and the dimensionless variable (Fig. 8)

$$t_P = \frac{T_P - T_W}{T_M - T_W}$$

is a function of $z$ for any particular geometry. As shown in Fig. 8, it does not depend significantly on the boundary conditions.

The value $t_P$ does not depend significantly on non-Newtonian flow index $n$ and the value for $\Psi \rightarrow 0$ is applicable even for finite values $\Psi$.

4. Conclusions

4.1. Definition of $\alpha$

The exponent $\alpha$ satisfying the relation (1) at different conditions for common heat exchangers usually assumes values close to the classical value 0.14. Actually, $\alpha$ is a function of the dimensionless length, $z$, and the scale $\Psi$ of viscosity changes, and of the non-Newtonian flow index, $n$. It depends on the boundary conditions, on the type of Nusselt number under consideration, and also somewhat on the definition of the characteristic ratio $\mu_W/\mu_0$. However, some of these variables have only minor effects on the viscosity correction.
4.2. Effect of the heat exchanger length, \( z \)

For different geometries (circular pipe and flat duct with asymmetric heat transfer) and for different boundary conditions (CWT and CHF) the exponent \( \alpha \) assumes finite values \( \alpha \approx 0.25 \) at \( z \rightarrow 0 \) and approximately half of this value at high \( z \). Generally, there are three distinct classes of practical heat transfer applications related to the studied problem:

(i) For very short contact with heat exchanging walls, say \( z < 10^{-5} \), we can use the asymptotic value \( \alpha \approx 0.25 \), which can slightly overestimate the viscosity effect. The value \( \alpha = 0.25 \) was determined from experimental heat transfer data in mixing vessels (Chapman et al. [16], Uhl [17]), and the short contact assumption is probably valid also for the heat transfer from submerged bodies.

(ii) The most frequently used heat exchangers utilize considerable part of the temperature difference between the inlet temperature \( T_0 \) and the wall temperature \( T_b \). It is usually sufficient to have about \( z \approx 0.01 \) for reaching \( (T_0 - T_b)/(T_0 - T_b) = 0.1 \) (employing 10% of the temperature difference). For employing 90% of the difference, the necessary length should be about \( z = 0.5 \). Apparently, the range close to the value \( z = 0.07 \), where there is just a minimum on the function \( \alpha(z) \) belonging to the mean Nusselt number, is the most important one. This is precisely the range, where Sieder and Tate [1] did their experiments. Our theoretical results prove that they had measured accurately, because the theoretical values here of \( \alpha \) are \( 0.14 \pm 0.01 \).

(iii) The range of long contact with wall, \( z > 0.5 \), is typical for thermostating. Respective values of \( \alpha \) are here slightly higher, say \( \alpha = 0.15 \).

4.3. Effect of the viscosity ratio, \( \Psi \)

The effect of the Pearson number \( \Psi \) is not extremely significant here, and in many practical applications we can use one value for both heating (\( \Psi > 0 \)) and cooling (\( \Psi < 0 \)).

4.4. Effect of non-Newtonian flow index, \( n \)

The value \( \alpha \) for non-Newtonian liquids depends also on \( n \). Suggested definition assures that \( \lim_{z \to 0} \alpha(n) = \lim_{z \to \infty} \alpha(n) \). For non-Newtonian liquids at large \( z \) we have lower values \( \alpha_X \), and the empirical rule \( \alpha \approx 0.14n \), suggested by Metzner et al. [6] has some validity.

4.5. Effect of geometry

Among different cross sections of heat exchangers, just the circular tube and a flat channel with asymmetrical heat transfer represent the extremes. However, the respective courses of \( \alpha(z) \) are very closed which means that the effect of geometry is less significant here.

4.6. Friction

Actual friction in non-isoviscous flow can be calculated as the friction of the isothermal flow at the pressure-averaged temperature \( T_P \). The value \( T_P \) is within the interval between \( T_W \) and \( T_M \) and can be predicted from Fig. 8.

Appendix A. Nomenclature

\begin{itemize}
  \item A coefficient (Eqs. (23) and (24))
  \item B characteristic linear dimension (Fig. 1)
  \item c\_p specific heat
  \item h heat transfer coefficient
  \item K consistency coefficient
  \item n non-Newtonian flow index
  \item Nu Nusselt number (Eq. (15))
  \item R tube radius
  \item T temperature
  \item t dimensionless temperature (Eq. (25))
  \item U mean velocity
  \item W flat channel width
  \item x distance from beginning of the exchanger
  \item z dimensionless axial coordinate (Eq. (12))
\end{itemize}

Greek letters

\begin{itemize}
  \item \( \alpha \) Sieder and Tate style exponent (Eqs. (7), (10) and (11))
  \item \( \gamma \) shear rate
  \item \( \eta \) consistency coefficient (Eq. (13))
  \item \( \lambda \) thermal conductivity
  \item \( \mu \) viscosity
  \item \( \rho \) density
  \item \( \tau \) shear stress
  \item \( \Psi \) Pearson number (Eqs. (6) and (14))
\end{itemize}

Indexes

\begin{itemize}
  \item 0 initial conditions
  \item B bulk
  \item CHF constant wall heat flux
  \item CWT constant wall temperature
  \item M mean
  \item P related to friction
  \item ST Sieder and Tate
  \item W wall
  \item X local
  \item * isothermal condition
References