FREE LEVEL EFFECT ON THE IMPELLER POWER INPUT IN BAFFLED TANKS

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Analysis of extended data on turbine impeller power input in geometrically similar agitated baffled tanks shows that the power number $Po$ is a function of Reynolds number $Po = Po(Re)$ until the emergence of surface aeration. Though it is usually anticipated that $Po = \text{const}$ in high Reynolds number region, some, whatever weak, function should be taken into consideration in more detailed analysis of the power data even here. In practice, disturbances of level and gas captured in the impeller region play also a significant role, namely in smaller tanks at higher impeller speeds. Decrease of power input can be explained by decrease of gas–liquid mixture density, or in other words by increase of efficient gas holdup $\varepsilon_E$ just in the impeller region. The value $\varepsilon_E$ defined by the relation $Po = Po(Re)/(1 + \varepsilon_E)$ was determined from the available data. Like other effects of the surface aeration it depends mainly on the dimensionless number $Nc = (We Fr)^{1/4}$. A simple correlation $\varepsilon_E (Nc)$ is suggested as a correction factor for prediction of impeller power in presence of gas capture.

The power transferred to the agitated liquid is a basic integral quantity characterizing processes in agitated vessels. Knowledge of the power input $P$ itself is important for the design of mechanical parts of the mixing equipment. Moreover, the amount of mechanical energy dissipated in the liquid characterizes in some manner the stirring contribution to any mixing process. Therefore, the value of specific power, that is the power per unit volume is an important criterion that can be employed to compare the intensity of stirring in mixing equipment of different types and sizes. The third reason for interest in impeller power is the experience that torque can be used for diagnostics of process regimes such as solid suspension, phase inversion in liquid–liquid dispersions, or aeration.

Impeller power can be expressed as a dimensionless power number $Po = P/\rho N^3 d^5$. For finite values $We$ and $Fr$ the liquid level in the tank may be disturbed, and generally we can assume for agitation of Newtonian liquids in geometrically similar equipment

$$Po = f(Re, Fr, We) .$$  

In many agitated systems the liquid level is essentially flat as a result of dominant gravity forces, $Fr \to 0$, or dominant surface tension, $We \to 0$. Under such circumstances the power number is a simple function of Reynolds number,

$$Po = Po^*(Re).$$

While effect of $We$ to the impeller power has not been considered explicitly, certain investigations of $Fr$ in agitated vessels were published. Rushton et al., who collected an extended set of power data for Newtonian liquids, presented the power number as a function of $Re$ for various impellers, and concluded that there is no effect of $Fr$ on power consumption in baffled vessels. Later, Nagata discovered that the effect of $Fr$ in Rushton’s data for unbaffled vessels is mostly a result of misinterpretation of errors in torque measurements.

The function $Po^*(Re)$, the power characteristics, is specific for any shape of mixing equipment. In the literature we can find the power characteristics for common mixing equipment, and the contribution of particular geometry parameters to their course. Practical application of open impellers such as turbines or propellers is limited to the range $Re > 10^3$, where the inertial forces remain more important than the viscous forces and therefore it is believed that there may exist an asymptotic constant value

$$\lim_{Re \to \infty} Po^*(Re) = Po_\infty.$$ 

The complete shape of the function $Po^*(Re)$ is simply a function of geometry simplexes of the agitated vessel. It can be determined experimentally provided that the measurement is carried out under the condition of undisturbed level to preserve the geometric similarity also here. Recently, Bujalski et al. pointed out on the basis of their own precise experiments with a set of geometrically similar mixing equipment of different size, and on the basis of certain data of Wisdom, Chapman et al., and Strek et al., that the value of $Po_\infty$ also depends on the equipment scale. Such a conclusion is a contradiction of the similarity theory and by our opinion it is result of neglecting the surface aeration at violent agitation. This hypothesis is tested in presented paper.

**EXPERIMENTAL DATA**

**Universal Power Characteristics**

Impeller power inputs were taken from the data measured by Sverak for four-blade impellers in a set of 10 sizes of tanks within the range of diameters $D = 0.0576-1.00$ m, and partly reported by Sverak and Hruby. Main geometry simplexes were $d/D = 1/3$, $w/d = 1/5$, $H_2/D = 1/3$, and geometric...
cal similarity were strictly preserved, including also less important quantities as thickness of the impeller blades, shaft and hub diameters, baffle thickness etc. The measurements were carried out not only for water, but also for liquids with different densities, and viscosities: glycerine solutions 91%, 74%, 44%, 25% in water, carbon tetrachloride, ethyl iodide, and mercury.

Individual power curves \( P_0(Re) \) were obtained by measuring the torque for a given impeller system with a given liquid at different rotation speeds. Simultaneously with the power measurement, the flow regime in the agitated liquid was observed. With increasing rotation speed, the free liquid surface stays a little disturbed, and for any individual case some threshold rotation speed, \( N_{JA} \), was found, corresponding to the "just aerated" state when first bubbles are regularly drawn down to the impeller. Figure 1a shows the individual power curves for water and for 91% (by weight) glycerine in geometrically similar agitated tanks of different sizes. Figure 1b shows the individual power curves as obtained in one mixing equipment with different liquids. However, when only the data evidently unaffected by the aeration are considered, as plotted in Fig. 1c, we obtain a single universal power characteristics \( P_0^*(Re) \) for the whole range \( Re \in (30 - 2 \cdot 10^5) \).

**RESULTS AND DISCUSSION**

**Surface Aeration: Gas Holdup and Impeller Power**

Significant decrease of the slope of individual power curves which appears at higher \( Re \) can be explained by presence of gas bubbles captured in the impeller region. Presented power data exhibit under the conditions of surface aeration a systematic decrease of \( P_0 \) with respect to \( P_0^*(Re) \). In the range, where the impeller power consumption is controlled by the kinetic energy, it can be explained by decreasing density of gas–liquid mixture due to the holdup of gas just in the impeller region. Actual local gas holdup can hardly be measured directly but it is easy to determine the efficient density, \( \rho_E \), and the efficient holdup, \( \varepsilon_E \), defined on the basis of power data by relations

\[
1 + \varepsilon_E = \rho/\rho_E = P_0'/P_0 . \tag{4}
\]

The efficient holdup in the impeller region \( \varepsilon_E \) is considerably higher, approximately four times, than the average holdup in agitated vessels determined from the level elevation. Our hypothesis is that it is just the ratio of the characteristic liquid downstream velocity and terminal bubble velocity, which controls transport of bubbles from the surface waves to the impeller and the gas holdup. The liquid circulation in low viscosity liquids is proportional to the product \( Nd \), and the bubble rise velocity is proportional to \((\sigma g/\rho)^{1/4}\) (see Peebles and Garber). Therefore we assume that the process is characterized by the value of the recirculation number

\[
N_c = Nd (\rho/\sigma g)^{1/4} = (WeFr)^{1/4} . \tag{5}
\]
Fig. 1
Individual power characteristics for: a 4-blade impellers in water and glycerine for different impeller diameters: □, ○ data by Sverak, + data by King et al., and b 4-blade impeller d = 36.8 mm in different liquids (data by Sverak): ○ glycerine 91%, ● glycerine 74%, ◆ glycerine 44%, ◆ glycerine 25%, ■ water, ▲ carbon tetrachloride, △ ethyl iodide, Δ mercury (d = 30 mm). c Power numbers Po = Po* below the onset of surface aeration (data by Sverak): ○ 4-blade paddle, ● Rushton turbine.
Westerterp et al.\textsuperscript{10} found that the average holdup in surface aerated vessels is a function of either \( N_c \) (for electrolytes) or \( Fr \) (for pure liquids). However, through the statistical examination\textsuperscript{12} of the data\textsuperscript{7} for pure liquids we proved that just aerated state for all the liquids and vessel sizes corresponds to a definite value \( N_{cJA} \). We have determined actual values of the critical aeration number, being about \( N_{cJA} \approx 8 \) for the four-blade impellers, and \( N_{cJA} \approx 4.5 \) for the Rushton turbines. The efficient holdup is practically zero for \( N_c < N_{cJA} \) and for \( N_c > N_{cJA} \) it increases with increasing \( N_c \). We are testing the hypothesis that \( \varepsilon_E \) is a function of \( N_c \) in Fig. 2. Though there is a considerable scatter of the data, no additional systematic trends depending on the liquid properties or equipment size has been found. It allows us to separate the effects of \( Re \) and \( N_c \) in the function \( (I) \) as

\[
Po = Po^*(Re)/(1 - \varepsilon_E(N_c))
\]  

(6)

For \( \varepsilon_E \) prediction we are suggesting to use a simple formula

\[
\varepsilon_E = 0 \quad \text{for } N_c < 1.4N_{cJA}
\]

\[
\varepsilon_E = 0.12(N_c - 1.4N_{cJA}) \quad \text{for } N_c > 1.4N_{cJA}
\]  

(7)

which predicts for particular impellers the power decrease by the curves in Fig. 2. Loss of stability and oscillations of the impeller power appear at high values of \( N_c \) where \( \varepsilon_E > 1 \).
We have also examined some data for turbine impellers in unbaffled tanks, given by Tetamanti et al.\textsuperscript{13} and Blasinski et al.\textsuperscript{14}, and we have found that the power decrease at higher \( N_c \) can be interpreted also in that case by the formula \((7)\) with nearly the same value \( N_{c,JA} \).

**Ultimate Power Number and Its Scale Dependence**

Existence of the universal power characteristics \( P_0^*(Re) \) for given mixing equipment is apparent from Fig. 3, where the power data affected by the surface aeration have been excluded. The function \( P_0^*(Re) \) for radial impellers in baffled vessels is non-decreasing for high \( Re \) usually. Significant decrease of the power beyond the value \( N_{c,JA} \) implies that in the vessels with open liquid level, any individual function \( P_0(Re) \) has a local maximum close to the point \( Re_{i,max} \approx N_{c,JA}(d\rho\gamma g)^{1/4}/\mu \). The respective value \( P_{0,i,max} \approx P_0^*(Re_{i,max}) \) may slightly depend both on the equipment scale and liquid properties, as noted by Sverak and Hruby\textsuperscript{8}.

Presented universal power characteristics increases slowly even at \( Re = 2 \cdot 10^5 \), and we cannot detect any existence of the constant asymptote \((3)\). The similarity theory says that if \( P_\infty \) exists, it must not be scale-dependent. If several authors\textsuperscript{3-6} declare that the ultimate value depends slightly on the scale, they are probably speaking just about the individual stable values \( P_{0,i,max} \).

**CONCLUSIONS**

The assumption that the power number for Newtonian liquids with free level in geometrically similar equipment is a function of \( Re \) with a minor effect of \( Fr \) is insufficient; the effect of captured bubbles is crucial during vigorous agitation, and \( We \) should be also included into the dimensional analysis.

If the level of agitated liquid is essentially undisturbed, then \( P_0 = P_0^*(Re) \). Though the function \( P_0^*(Re) \) is very weak for \( Re > 100 \), the existence of ultimate value \( P_{0,\infty} \) anticipated by Rushton\textsuperscript{1} has not been confirmed. In literature we can find some experimentally evaluated values declared as \( P_{0,\infty} \) for particular mixing equipments. Examination of these data indicates that those values are actually the individual stable value \( P_{0,i,max} \) for chosen liquid in chosen size of mixing equipment. Therefore, we must not be surprised that such a quantity is scale-dependent.

A typical regime when the process in an agitated vessel is affected by the level phenomena is the regime of surface aeration. Its significance is measured by the value of the recirculation criterion \( N_c = (Fr We)^{1/4} \). The effect of surface aeration on the impeller power can be separated from the effect of \( Re \) and the power number correlation can be expressed by superposition \((6)\) of the function \( P_0^*(Re) \), and the efficient holdup function \( \varepsilon_E(N_c) \).

SYMBOLS

d \quad \text{impeller diameter, m}

D \quad \text{tank diameter, m}

g \quad \text{gravity acceleration, m s}^{-2}

H_2 \quad \text{impeller to bottom clearance, m}

N_0 \quad \text{impeller speed, s}^{-1}

P \quad \text{impeller power input, W}

w \quad \text{impeller blade width, m}

\varepsilon \quad \text{gas holdup, m}^3 \text{m}^{-3}

\mu \quad \text{liquid viscosity, Pa s}

\rho \quad \text{liquid density, kg m}^{-3}

\sigma \quad \text{surface tension, N m}^{-1}

Fr \equiv \frac{Nd^2}{g} \quad \text{Froude number}

Po \equiv \frac{P}{(\rho N^3 d^5)} \quad \text{power number}

Po^* \quad \text{power number for systems unaffected by aeration}

Ne \equiv \frac{Nd(\rho/\sigma g)^{1/4}} \quad \text{recirculation number}

Re \equiv \frac{Nd \rho \mu}{\sigma} \quad \text{Reynolds number}

We \equiv \frac{N^3 d^2 \rho \sigma}{\sigma} \quad \text{Weber number}

Subscripts

E \quad \text{efficient value}

JA \quad \text{just aerated state}

i \quad \text{individual value for a given liquid and impeller}

\text{max} \quad \text{maximum value}

\infty \quad \text{asymptotic value for high } Re

REFERENCES