Periodical shape and velocity oscillations of rising ellipsoidal bubbles

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Abstract
Experiments with rising bubbles levitating in downward liquid flow enable us to do long-term observation of the bubble motion. Medium size ellipsoidal bubbles have been observed systematically and extended set of data on oscillations of their shape and velocity is evaluated statistically in this paper. The bubble motion is wobbling and their path is zig-zag or slightly helical. Simple hydrodynamics model is suggested to explain this behavior qualitatively and to predict frequency and amplitude of the oscillation quantitatively. Typical values for bubbles that range in diameter from 2 to 20 mm are rising velocities of 200-300 mm/s, frequency of oscillations of 5-6 Hz, and maximum drift velocities of 150 mm/s. Particular results of experiments and CFD computations available in literature confirm suitability of the presented model.

Keywords: rising ellipsoidal bubble, oscillation, drift, wobbling, autocorrelation

1. Introduction
Study of the motion of a single bubble in liquid at rest is a first step in understanding complex problems of gas – liquid systems in process technologies. Trajectory of larger bubbles rising in liquids is not a straight vertical line. It has apparently helical or zig-zag pattern (Fig.1). Prospetti (2004) noted that such behavior had been mentioned in the notebooks of Leonardo da Vinci (1515) and introduced the term „Leonardo’s paradox“.


In parallel, realistic hydrodynamics models including the case of deformable bubbles are solved by CFD methods e.g. Ryskin and Leal (1984), Bunner and Tryggvason (1999), Esmaeeli and Tryggvason (1999), Magnaudet and Eames (2000), Tomiyama (2002), Koebe (2004). However, the computation required by these models is time consuming and only specific results are available, in certain cases still affected by initial conditions of the model.

This paper is based on the experiments in countercurrent liquid flow (Wichterle et al.(2000, Vasconcelos et al. 2002), which enables long term observation of single bubble. Ellipsoidal bubbles of equivalent diameters 4 – 10 mm are the focus of our interest. This is the most common size of bubbles in equipment where coalescence and breakup takes place (Wichterle et al. 2005). Extended camera records of moving bubbles are treated by image analysis and the obtained data and handled statistically to characterize the bubble shape and velocity oscillations by a small number of parameters.

We have also suggested following simple mathematical model of the bubble motion.
2. Theoretical

2.1. Vertical forces on a bubble

Several forces should be taken into consideration. From the bubble buoyancy, \( \rho_L \) \( g V_B \), and gravity \( \rho_G \) \( g V_B \) (which is usually negligible) results the mass force oriented upwards

\[
F_G = (\rho_L - \rho_G) \ g V_B = \Delta \rho \ g V_B = \rho_L \ g V_B . \quad (1)
\]

In the opposite direction acts the hydraulic resistance. Its contribution on given front area \( S_F \) is usually expressed with help of drag coefficient, \( C_B \):

\[
F_R = C_B \ \frac{\rho_L \ u_B^2}{2} \ S_F , \quad (2)
\]

The drag coefficient \( C_B \) for creeping flow of small bubbles in high-viscosity liquids can be approximately predicted by Stokes law. It decreases when Reynold's number increases. For medium size bubbles in low viscosity liquids (typical for common air bubbles in water), \( C_B \) is nearly independent of Reynold's number and usually \( 0.5 < C_B < 1 \). The Reynold's number for bubbles

\[
Re = \frac{d_B \ u_B \ \rho_L}{\mu_L}, \quad (3)
\]

is based on the mean rising velocity \( u_B \) and the diameter of equivalent sphere

\[
d_B = \left( \frac{6 V_B}{\pi} \right)^{1/3}. \quad (4)
\]

Both the buoyancy and drag forces depend on the bubble shape that is controlled mainly by surface and gravity forces. The surface force

\[
\frac{\Delta F_S}{\Delta S_B} = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \quad (5)
\]

acts normally to any surface element \( \Delta S_B \) of curved interfacial surface and depends on two main curvature radiiueses \( R_1 \) and \( R_2 \). Its importance for bubbles is expressed by Eötvös number

\[
Eo = \frac{d_B^2 \ \Delta \rho \ g}{\sigma} \quad (6)
\]

For \( Eo << 1 \), the bubbles are essentially spherical; for air bubbles in water it is relevant when \( d_B < 2 \text{ mm} \). Larger bubbles are deformed; for \( 1 < Eo < 40 \) their shape is similar to an oblate ellipsoid with its minor axis oriented vertically.

The classical formula for hydraulic drag \( C_B \) was developed for solid bodies of a known front area. For bubbles and drops we prefer a modified drag coefficient, \( C \), related to the known front area of the equivalent sphere,

\[
S_B = \frac{\pi}{4} \ d_B^2 \quad (7)
\]

and

\[
F_B = C \ \frac{\rho_L \ u_B^2}{2} \ S_B \quad (8)
\]

The dimensionless function \( C(Re) \) can be determined from average terminal velocity \( u_B \) on the bubble size \( d_B \) providing \( F_G = F_R \). This function within a certain range of \( Re = F_R \) is significantly effected by interface mobility, which may be blocked when surface active compounds are present in contaminated liquids. In Fig.2, the range delimited by Clift, Grace and Weber (1978) from previous experimental data is presented. Recently Tomiyama (2002b) has discovered that the lower limit of drag may be the effect of unsteady slender bubble shape just after its formation while the developed bubble rising approached the upper limit of \( C \).

2.2. Bubble acceleration

Bubble acceleration depends on the sum of external forces, \( F \)

\[
\frac{du}{dt} = \frac{F}{m_v} \quad (9)
\]

and the virtual mass \( m_v \) includes the bubble mass and the mass of the ambient liquid moving simultaneously with the bubble

\[
m_v = (\rho_G + C_M \rho_L) \ V_B = C_M \ \rho_L \ V_B \quad (10)
\]

The usual estimate of the coefficient of virtual mass for bubbles is

\[
C_M \approx 0.5. \quad (11)
\]

With respect to different buoyancy forces to the thicker and thinner sides of the bubble, upward acceleration on both sides differs and therefore the bubble wobbles. The pressure inside the bubble is practically uniform and then ambient hydrostatic pressure is balanced by surface tension.

\[
\alpha = \arcsin \left( \frac{h}{\frac{2}{a}} \right) \quad (12)
\]
Evidently, a modified number or a new parameter can be assumed in order to study the bubble inclination. According to the experimental data \( \alpha_0 = \pi/4 \), bubble inclination \( \alpha \) oscillates in the range from \(-\alpha_0\) to \(+\alpha_0\). Angular velocity of this motion is 
\[
\omega = 2\pi f \alpha_0 \cos(2\pi f t) \quad (23)
\]

Such a rotation induces a drift force \( F_D \) oriented toward the direction where difference of surface velocity and ambient velocity is smaller. The problem was solved for solid spheres (Yu, Phan-Thien and Tanner 2004, Bagchi and Balachandar 2002) where potential flow theory predicts the drift force as
\[
F_D = \rho_L V_B \omega \times u_B . \quad (24)
\]

This force causes drift movement of the particle perpendicular to the main flow (Fig. 4).

When the induced drift velocity is \( u_D \), it is retarded by drag force
\[
F_{HD} = C_D \frac{\rho_L}{2} u_D^2 S_B . \quad (25)
\]

As the drift velocity \( u_D \) is low and oscillates around zero, the respective Reynolds number is low as well and Stokes law
\[
C = \frac{24 \mu}{d_B} \frac{u_D}{\rho_L} \quad (26)
\]
is acceptable for spherical bodies. Two force components \( F_S \) and \( F_{HD} \), responsible for the drift of a sphere rotating in an oscillatory way according to Eq.(23) change the drift momentum so that
\[
C_M \rho_L V_B \frac{d u_D}{d t} = \frac{12 S_B \mu}{d_B} u_D \quad (27)
\]

Solution of this equation can be expressed by using the dimensionless parameter
\[
R = \frac{\pi C_M d_B^2 \rho_L}{9 \mu} \quad (28)
\]
which (like Reynolds number) relates inertial force of sphere drift to viscous forces. By integration of Eq. (27) the drift velocity becomes

\[ u_D = \frac{1}{\sqrt{1 + (1/R^2)}} \frac{u_B \alpha_0}{C_M} \sin(2\pi f t - \arctan(1/R)) \]

(29)

Its maximum value is of the same order as the rising velocity \( u_B \). It means that drift affects significantly the bubble path. There is no preference for the drift direction; the presented theory predicts a zig-zag motion in any direction. However, the bubble may also wobble in two directions with a certain time shift and then the path might be helical.

Further integration of Eq.(29) predicts horizontal position, \( x \), in the drift direction as

\[ x = \frac{-1}{\sqrt{1 + (1/R^2)}} \frac{u_B \alpha_0}{2\pi f C_M} \cos(2\pi f t - \arctan(1/R)) \]

(30)

If the viscosity effect is negligible, \((1/R) \rightarrow 0\) and the expression can be simplified. On the other hand, the finite value \(1/R\) (the effect of viscosity) decreases the drift amplitude and causes phase shift of drift with respect to the bubble oscillation.

The presented drift model enables us to elegantly explain the nature of Leonardo’s paradox – why are larger rising bubbles oriented to the direction of motion by their larger front area and why their path is zig-zag or helical. This is essentially different than motion of flat solid bodies.

3. Experimental

Herein bubbles are studied in downward flow of liquid in a divergent conical channel (diameter 50-70 mm). The channel is arranged as an entrance from a large calming vessel; therefore the liquid velocity profile outside the boundary layer at the walls is quite flat with a low turbulence level. Detailed description of the equipment was presented by Wichterle et al. (2000 and 2005). The channel is placed in a square PMMA vessel 100x100x300 mm. Using a camera, placed in distance 2 m from the object, we monitored two projections of bubbles, one direct and one mirror image (Fig.5). In this paper we present experiments with bubbles 50-500 mm³ in non-filtered de-ionized water (which is probably contaminated by surface active impurities). Smaller bubbles are essentially spherical and their rising was extensively studied by other authors. Larger bubbles cannot be observed for longer period due to their breakup (Wichterle et al., 2005).

Using a standard video camera (25 Hz) we obtained sequences rendered as *.avi file, which were treated off-line by image analysis through IMAQ software, in environment LabView. Selected data were recorded by high speed camera Kodak (125 or 250 Hz was suitable frequency). The frames were stored as separated *.bmp files and then combined to *.avi files.

In all cases, the results were time series of quantities related to the position, size, and shape of bubbles as a Excel sheet *.xls.

Selected quantities were:
- left border in front projection
- right border in front projection
- mass center in front projection
- left border in mirror projection
- right border in mirror projection
- mass center in mirror projection
- upper border
- lower border
- vertical position of mass center
- major axis of equivalent ellipse in front projection
- minor axis of equivalent ellipse in front projection
- major axis of equivalent ellipse in mirror projection
- minor axis of equivalent ellipse in mirror projection
- bubble inclination in front projection
- bubble inclination in mirror projection
- bubble area in front projection
- bubble area in mirror projection

5. Results

The positions of bubbles were recalculated from pixels to millimeters.

Vertical position of bubbles for a given channel and volume flow rate give rising velocity, \( u_Z \), and its mean value, \( u_B \).

Difference of mass center position enables the horizontal components \( u_X \), \( u_Y \) of velocity, drift velocity, \( u_D = (u_X^2 + u_Y^2)^{1/2} \), and drift acceleration, \( \alpha_Z \) to be calculated.

5.1. Frequency

Any quantity \( v \) is oscillates around a mean value

\[ \bar{v} = \frac{1}{n} \sum_{i=1}^{n} v_i . \]

When a time series \( n \) points \( v_i \) is chosen, the autocorrelation function is the series

\[ A_k = \frac{\sum_{i=1}^{n} (v_i - \bar{v})(v_{i+k} - \bar{v})}{\sum_{i=1}^{n} (v_i - \bar{v})^2} \]

(32)

For periodic functions the distance of local maxima of the autocorrelation function (taken as a continuous function) correspond to the time of one period. It is used for determination of frequency, \( f \). The
autocorrelation function is not a pure cosine function and is damped with increasing $k$ as a result of random variation of the variable.

Examples of autocorrelation functions of various quantities for two sizes of bubbles ($75 \text{ mm}^3$ and $300 \text{ mm}^3$) are plotted in Fig. 6. Obtained characteristic frequency is plotted in Fig. 7.

Our data indicate a local minimum $f$ close to $d_B \approx 7.5 \text{ mm}$. The set of data in Fig. 7 obtained by different authors and different techniques do not allow us to indicate detailed trends and we can only conclude that characteristic frequencies lies within limits 4-7 Hz for a wide range of bubbles size.

In Fig. 9., the same data set is plotted in dimensionless form as function $St_L(Eo)$. Even here we can see some variation and we can conclude that for $0.1 < Eo < 50$ is $St_L \approx 0.5 \pm 0.2$. Few results for diluted glycerol in water ($\mu = 3.38 \text{ mPas}$) indicate that the effect of viscosity is less significant.

Besides the frequency of bubble oscillation, the autocorrelation indicates other frequency around 0.5 Hz. This is probably the frequency of liquid column rotation due to Coriolis force at the entrance of liquid to the measuring section; this rotation is in any case counterclockwise in upper projection.

Some authors have determined slightly higher frequencies of single bubble motion immediately after its release from capillary. Wu and Gharib (1998) determined 12 Hz in a short water column. Tomiyama (2002b) pointed out that bubble oscillation stabilization is slower in water free of surface active impurities. These data are not involved in Figs. 8 and 9.

5.2. Upper projection of the trajectory

Due to the fact that there exists a single characteristic frequency, the upper projection of the trajectory may be elliptical with major axis $2A$ and minor axis $2B$. If $A \approx B$ the bubble rising is helical. If $A >> B$ the bubble path is essentially zig-zag. Generally, the drift velocity is

$$ u_D = \sqrt{u_X^2 + u_Y^2} $$

and it depends on the frequency $f$ and time $t$:
6. Conclusion

1. The first model (17) of forces acting in a vertical direction to the bubble is suggested for ellipsoidal bubbles in low viscosity liquids (1<\( \varepsilon \omega <40 \), \( Re>100 \)). Its solution leads to following results:
   a. Oscillatory wobbling of the bubble is caused by non-linear compensation of external forces by surface tension.
   b. By integration of the model, characteristic frequency of wobbling (Eq.21) is obtained. Frequency of wobbling depends on the rising velocity and material properties; the effect of bubble size is minor.

2. The second model (27) is suggested to estimate the effect of horizontal forces acting to the wobbling bubbles. From its solution we can conclude:
   a. The horizontal drift is related to the bubble wobbling and its frequency is identical. Its velocity is comparable with the rising velocity.
   b. By Eq.(30) the amplitude of horizontal drift decreases with increasing viscosity and increases with bubble size. Phase shift of drift with respect to wobbling increases with viscosity.

3. Bubble position in downstream divergent liquid flow were recorded by high speed camera. Ellipsoidal bubbles of volumes 50 – 400 mm\(^3\) in water or glycerol solutions were studied.

4. By autocorrelation analysis of horizontal trajectories of bubbles and of their wobbling observed in two projections, characteristic frequencies were determined, which were in accordance with the model’s prediction (21).

5. Statistical analysis of the horizontal bubble velocity and its acceleration revealed the most probable ellipsoidal pattern of drift motion. Its amplitude is viscosity dependent as predicted by Eq.(30). For the medium size bubbles under investigation, the drift pattern is essentially zig-zag, while the helical pattern is more probable for smaller ones.

6. Adequacy of the model was tested by comparison with particular experimental and CFD data available in the literature.
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Notation

- \(a\): main semiaxis of the bubble side projection (Fig.3)
- \(a_p\): horizontal drift acceleration
- \(A\): major semiaxis of the trajectory upper projection
- \(A_t\): autocorrelation function
- \(B\): minor semiaxis of the trajectory upper projection
- \(C\): drag coefficient for spheres (8)
- \(C_B\): drag coefficient for bubbles (3)
- \(C_M\): virtual mass coefficient (10)
- \(d_B\): equivalent diameter of bubble (5)
- \(E_o\): Eötvös number (6)
- \(f\): frequency of bubble oscillations
- \(F_B\): buoyancy force (14)
- \(F_D\): drift force (23)
- \(F_{tt}\): horizontal hydraulic resistance (24)
- \(F_{tu}\): vertical hydraulic resistance (15)
- \(g\): gravity acceleration
- \(h\): bubble inclination (Fig.3)
- \(m_v\): virtual mass (10)
- \(r\): bubble curvature radius (Fig.3)
- \(R\): drift parameter
- \(R e\): Reynolds number (4)
- \(S_f\): bubble front area
- \(S_B\): equivalent bubble front area (7)
- \(S_t\): modified Strouhal number (21)
- \(t\): time
- \(u\): velocity
- \(u_B\): bubble rise velocity
- \(u_D\): horizontal drift velocity
- \(V_B\): bubble volume
- \(x\): horizontal coordinate
- \(\alpha\): bubble inclination (Fig.3)
- \(\alpha_0\): maximum value of \(\alpha\)
- \(\mu\): liquid viscosity
- \(\Delta \rho\): density difference
- \(\rho_l\): liquid density
- \(\sigma\): surface tension
- \(\omega\): angular velocity (22)

References


