Shape and rising velocity of bubbles

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Common medium-size bubbles that are produced by breakup of gas stream (for air-water system volume of such bubbles is 10-500 mm$^3$) are mostly like oblate ellipsoids and their rising path is somewhat helical. Correlations presented in this paper are based on records of rising bubbles, obtained using a high speed video camera. Shape of a bubble and its velocity was determined from time series of characteristic parameters of the bubble projection obtained by image analysis. For this purpose, we have developed a mathematical model of the bubble image including three parameters: eccentricity of the ellipsoid, amplitude of the wobbling, and angular rotation. For different low- and medium-viscosity liquids, eccentricity of the ellipsoids approximating the bubble shape is a function of Eötvös number, viscosity effect being negligible. On the other side, the bubble rising velocity depends both on viscosity and surface tension. However, the effect of surface tension may be included to the estimate of bubble front area and the modified drag coefficient is a simple function of the Reynolds number.

Introduction

Bubbles rising in liquid may become various shapes. In ideal case, the bubble of volume $V_B$ should be spherical and its diameter

$$d_B \equiv \sqrt[3]{\frac{6V_B}{\pi}} , \quad (1)$$

is generally called diameter of the equivalent sphere. It is well known that only small bubbles are spherical. In this paper, we are turning our attention to the most common medium size of rising bubbles (in water the air bubbles in the range 1 mm $< d_B < 15$ mm). Their shape is assumed to be like oblate ellipsoids with semiaxes $a$ and $b$. Path of a rising ellipsoidal bubble is either helical or zig-zag and the bubble is generally oriented by its minor axis to the direction of motion. The largest bubbles loss the fore aft symmetry and finally become spherical cap shape with hollow part at the bottom side. Their motion is more chaotic and they are vulnerable to spontaneous breakup.

Shape of ellipsoidal bubbles can be characterized simply by the aspect ratio $b/a$. In the fundamental textbook by Clift et al. (1978) appeared the recommendation to accept the correlation by Wellek et al. (1966) as suitable for rising bubbles. Actually, Wellek had analyzed photographs of rising drops of immiscible liquids in continuous liquid. Drop image were generally like an ellipse with the minor axis oriented vertically. Ratio of the drop horizontal width $2a$ to its vertical height $2b$ was interpreted by simple formula

$$\frac{a}{b} = 1 + 0.163Eo^{0.757} \quad \text{for} \quad Eo < 40 \quad (2)$$

where the Eötvös number is

$$Eo \equiv \frac{d_B^2 \Delta \rho g}{\sigma} . \quad (3)$$
\( \Delta \rho \) is density difference of continuous and dispersed phase and \( \sigma \) is the interface tension.

Apparently, effects of viscosity were abandoned here and later, numerous authors tried to improve the correlations including some from several other dimensionless numbers. One option is to use second argument, either the Morton number

\[
Mo \equiv \frac{\mu^4 g}{\sigma^3 \rho}
\]

(4)

or the Reynolds number

\[
Re \equiv \frac{d_B u_B \rho_L}{\mu_L}
\]

(5)

(\text{where } u_B \text{ is the bubble rising velocity}).

According to the map of rising bubble regimes in coordinates \((Eo, Re)\), presented by Clift et al., the effect of \( Re \) is unimportant for low viscosity liquids. Obviously, it should be taken into consideration in highly viscous and non-Newtonian liquids.

Welek et al. themselves tried to correlate data alternatively by using

\[
\frac{a}{b} = f(We)
\]

(6)

where the Weber number

\[
We \equiv \frac{d_B u_B^2 \rho_L}{\sigma}
\]

(7)

reflects also some viscosity effect. Nevertheless they stated that the correlation of the data by (6) was a little worse than (8).

\[
\frac{a}{b} = f(Eo)
\]

(8)

Nevertheless, other authors prefer the correlation (6) for bubbles.

Some compromising approach was adapted by Tadaki, using so called Tadaki number

\[
Ta \equiv Re Mo^{0.23} \approx Eo^{1/4} We^{1/2}
\]

(9)

Evidently, there is still considerable uncertainty in this matter, and new experimental data are required. In this paper, we are reporting the results obtained with ellipsoidal bubbles in unprepared water as a most important liquid medium, with the bubbles in glycerol solution (effect of viscosity), in butanol (effect of surface tension) and calcium chloride solution (effect of electrolytes).

**Experimental**

Herein bubbles are studied in downward flow of liquid in a divergent conical channel (diameter 50-70 mm). The channel is arranged as an entrance from large calming vessel; therefore the liquid velocity profile outside the boundary layer at the walls is quite flat with a low turbulence level. Detailed description of the equipment was presented by Wichterle et al. (2000 and 2005). The channel is placed in a square PMMA vessel 100x100x300 mm. Using a camera, placed in distance 2 m from the object, we monitored two projections of bubbles, one direct and one mirror image (Fig.1). In this paper we present experiments with bubbles 10-400 mm³ in non-filtered de-ionized water (which is probably contaminated by surface active impurities). Smaller bubbles are essentially spherical and have no interesting features. Larger bubbles cannot be observed for longer period due to their breakup (Wichterle et al., 2005).

Using a high speed camera Kodak (125 Hz was satisfactory frequency) we obtained sequences rendered as *.avi file, which were treated off-line by image analysis through IMAQ.
software, in environment LabView. The frames were stored as separated *.bmp files and then combined to *.avi files.

In all cases, the results were time series of quantities related to the position, size, and shape of bubbles as a Excel sheet *.xls. Any record covers 1363 frames of one rising bubble taken within the period 11 seconds.

Selected quantities were
- left border in front projection
- right border in front projection
- mass center in front projection
- left border in mirror projection
- right border in mirror projection
- mass center in mirror projection
- upper border
- lower border
- vertical position of mass center
- major axis of equivalent ellipse in front projection
- minor axis of equivalent ellipse in front projection
- major axis of equivalent ellipse in mirror projection
- minor axis of equivalent ellipse in mirror projection
- bubble inclination in front projection
- bubble inclination in mirror projection
- bubble area in front projection
- bubble area in mirror projection

Our first idea was to accept the parameters of equivalent ellipse as main characteristic of the bubble shape. These parameters are determined from the image area and perimeter. However, identification of smaller object’s perimeter is highly sensitive to any error in pixel assignment, and therefore the calculated eccentricity of equivalent ellipse can be significantly overestimated. Our procedure was based on the determined horizontal width, \( w \), and vertical height, \( h \), of the object.

**Calculated profiles of ideal oblate ellipsoids**

Shape of oblate ellipsoid in a standard position with the axis of symmetry (the minor axis) oriented vertically is apparent in Fig.3. for various values \( a/b \).

During the bubble rise, the angle \( \alpha \) between the vertical line and the minor axes oscillates in ideal case harmonically with amplitude \( \alpha_{\text{max}} \) and frequency \( f \)

\[
\alpha = \alpha_{\text{max}} \sin(2\pi f t)
\]

In upper projection, the bubble wobbling may be oriented to arbitrary angle \( \beta \) towards the projected area. This is obvious for helical rising, nevertheless, statistically it holds even for the essentially zig-zag pattern.

In the first row of Fig. 4 there are presented the profiles of ellipsoids with \( a/b=2 \) for \( \beta =0 \), with different values of \( \alpha \). Respective profiles of these ellipsoids rotating around the vertical axis are shown for \( 0 < \beta \leq \pi/2 \) in following rows of the Fig. 4. For other parts of interval \( <0 ; 2\pi > \), the profiles are either symmetrical or identical.

The oblate ellipsoid is projected to a vertical plane as an inclined ellipse \( k \). Angle \( \varphi \) of inclination for the ellipse \( k \) is

\[
\varphi = \arctan(\tan \alpha \cos \beta)
\]
When the bubble is observed from side in perpendicular direction, the bubble inclination angle $\phi_{90}$ in this view is

$$\phi_{90} = \arctan(\tan \alpha \sin \beta)$$

(9)

$$\beta = \arctan\left(\frac{\tan \phi_{90}}{\tan \phi}\right)$$

(10)

and

$$\alpha = \arctan\left(\frac{\tan \phi}{\cos \beta}\right)$$

(11)

Major semi-axis $A$ of the ellipse $k$ is simply

$$A = a$$

while the minor semi-axis $B$ is

$$B = \sqrt{b^2 + (a^2 - b^2)\sin^2 \alpha \sin^2 \beta}.$$  (12)

Rectangular circumscribed box for the ellipse $k$ has horizontal width

$$w = 2 \sqrt{A^2 \cos^2 \phi + B^2 \sin^2 \phi}$$

(13)

and vertical height

$$h = 2 \sqrt{A^2 \sin^2 \phi + B^2 \cos^2 \phi}.$$  (14)

Therefore,

$$\frac{A}{B} = \frac{a}{b} \sqrt{1 + \left(\frac{a}{b}\right)^2 - 1} \sin^2 \alpha \sin^2 \beta$$

(15)

Let us introduce

$$T \equiv \tan^2 \phi.$$  (16)

Then

$$\frac{w}{h} \leq \frac{A}{B} \leq \frac{a}{b}$$

(17)

Evidently, we have

$$\frac{w}{h} \leq \frac{A}{B} \leq \frac{a}{b}$$

(18)

Semiaxes of the characteristic ellipse approximating the projected bubble can be calculated from observed values $h$, $w$, and $\phi$

$$a = A = \frac{1}{2} \sqrt{\frac{w^2 - h^2 T}{1 - T}}$$

(19)

$$B = \frac{1}{2} \sqrt{\frac{h^2 - w^2 T}{1 - T}}$$

(20)

When the bubble is observed in the side view, respective quantities $w_{90}$, $h_{90}$, $h$, and $\phi_{90}$ are obtained. From this viewpoint the bubble image corresponds to the value $\beta_{90} = \beta + \pi/2$ instead of $\beta$. Therefore

$$B_{90} = \sqrt{b^2 + (a^2 - b^2)\sin^2 \alpha \cos^2 \beta}$$

(21)
By combination of (20) and (21) with
\[ T_{90} \equiv \tan^2 \phi_{90} \] (22)
and finally, using (13) and (14), the minor semiaxis of the oblate ellipsoid can be determined on the basis of observed data by
\[
b = \frac{1}{2} \sqrt{\frac{h^2 (1 + T_{90}) - \omega^2 (T + T_{90})}{1 - T}} \]
\[ = \frac{1}{2} \sqrt{\frac{h^2 (1 + T) - \omega^2 (T + T_{90})}{1 - T_{90}}} \] (23)
is a function of both \( \alpha \) and \( \beta \).

**Observed shape of bubbles**
The semiaxes \( a, b \) of the equivalent oblate ellipsoid have been calculated from experimental data. Any point plotted in the Fig.3 represents statistic evaluation of 1363 images of one rising bubble.

![Fig.3. Calculated semiaxes of the oblate ellipsoid approximating the bubble shape](image)

To check the effects of liquid properties to the bubble shape, we studied dimensionless geometry variable \((2a/d_b)^2\) which is just ratio of front area of an actual bubble to the one of the equivalent sphere. Therefore, it is important quantity controlling the drag force on rising bubbles. It is plotted in Figs 4a-c. Evidently, the Eötvös number is of a primary importance and correlations including the effect of viscosity and bubble velocity (\(Re\) or \(We\)) brought no advantage.
Fig. 4. Dimensionless front area of bubbles correlated with various criteria.
**Static bubble model**

With respect to the minor effect of bubble dynamics to the shape of ellipsoidal bubbles in low- and moderate-viscosity liquids, we can compare the shape of rising bubbles with some kind of static bubbles. Shapes of static bubbles can be numerically calculated by integration of Laplace-Young equation. Few examples of calculated profiles of axisymmetric bubbles under a wetted horizontal plate are shown in Fig.5. All lengths are normalized by so called Laplace length, i.e.

\[ L \equiv \frac{\sigma}{\Delta \rho g} \]

The shape of such a class of bubbles can be approximated by oblate ellipsoids characterized by semiaxes \( a_S, b_S \) estimated as

\[ a_S = x_{\text{max}} \]
\[ 2b_S = -y_{\text{min}} \]
The calculated values can be approximated by the empiric formula
\[ \frac{a_s}{b_s} = 1 + 0.185 Eo^{0.80} \quad \text{for} \quad Eo < 40 \] (25)
similar to the one by Wellek et al. The computed values \( b_s/L \) and \( a_s/L \) are also plotted in Fig. 6. together with the experimental data for rising bubbles.

### Rising velocity of contaminated bubbles

Rising velocity as a function of the bubble diameter is plotted in Fig. 7.

![Graph of rising velocity vs. diameter](image)

Fig.7. Rising velocity of bubbles in various liquids. Any point represents the averaged value of the

Evidently, the rising velocity is affected by density, viscosity and surface tension. Generally,

\[ u = f(d, g, \rho, \Delta \rho, \mu, \sigma) \]

For bubbles, we can assume \( \Delta \rho \approx \rho \), and therefore 3 dimensionless number can describe the process. When the effect of viscosity, \( \mu \), is ignored, two remaining numbers can be

\[ U_\sigma \equiv \frac{u}{W} = \frac{Re Mo^{\frac{1}{4}}}{Eo^{\frac{1}{2}}} \] (26)

(where \( W \equiv \left( \frac{\sigma g}{\rho} \right)^{\frac{1}{4}} \)) (27)

and

\[ D_\sigma \equiv \frac{d}{L} = \frac{d}{Eo^{\frac{1}{2}}} \] (28)
From the plot in Fig. 8 we can see that the effect of viscosity cannot be completely neglected. Other option is to ignore the effect of surface tension, $\sigma$. In such a case other dimensionless groups can be introduced by relations

$$U_\mu \equiv \frac{u}{\left(\frac{4\mu g}{3\rho}\right)^{\frac{1}{3}}} = \left(\frac{Re}{C_D}\right)^{\frac{1}{3}} \quad (29)$$

$$D_\mu \equiv \frac{d}{\left(\frac{\mu}{\rho}\right)^{\frac{2}{3}} \left(\frac{4g}{3}\right)^{\frac{1}{3}}} = Re^{2/3} C_D^{1/3} \quad (30)$$

The drag coefficient, $C_D$, is

$$C_D \equiv \frac{F}{\frac{\rho}{2} u^2 S} \quad (31)$$

For a steady state rising bubble, the drag resistance force, $F$, is just equal to the buoyancy

$$F = \Delta \rho g \frac{\pi}{6} d^3 \quad (32)$$

and front area, $(\pi d^2/4)$, of the equivalent sphere is used instead of $S$. Then

$$C_D = \frac{4dg \Delta \rho}{3u^2 \rho} \approx \frac{4dg}{3u^2} \quad (33)$$
Plot of $C_D$ versus $Re$ is presented in Fig. 9.

Data for butanol, having considerably lower surface tension does not match a single line and this interpretation is also unsatisfactory. Therefore, three criteria should be taken into consideration, e.g there is a function $U_\mu (D_\mu, D_\sigma)$.

It is desirable to separate, as possible, the effect of surface tension. From investigation of the shape of bubbles we have found that bubbles are oblate ellipsoids for higher $Eo$. Front area of the bubble, $(\pi a^2)$, can be used in definition of the modified drag coefficient,

$$C_A \equiv C_D \left( \frac{d}{2a} \right)^2 \approx \frac{4d}{3a^2} \left( \frac{d}{2a} \right)^2$$

and dimensionless velocity and diameter defined relations

$$U_{\mu} \equiv U_\mu \left( \frac{d}{2a} \right)^{-2/3}$$

$$D_{\mu} \equiv D_\mu \left( \frac{d}{2a} \right)^{2/3}$$

When a simple correlation suggested for bubbles

$$\left( \frac{2a}{d} \right)^2 = 1 + 0.095Eo^{0.75} \text{ for } Eo<20$$

is applied, quite successful correlation plotted in Fig. 10 is obtained.

The data in Fig. 10 simple are approximated by the power-law trendline

$$U_{\mu} = 1.6 \ D_{\mu}^{0.4}$$

With (38), complete correlation of velocity can expressed by using classical dimensionless variables as
This correlation fits well the data for medium size bubbles in contaminated low- and medium-viscosity liquids. In carefully prepared pure liquids, the rising velocity of can be slightly higher.

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**References**


